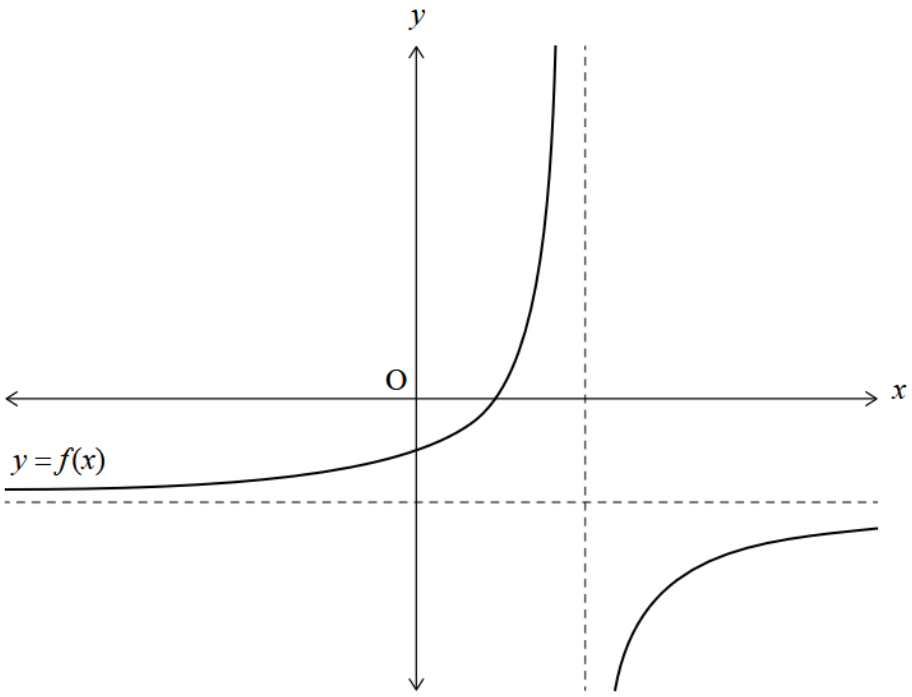
Rational Functions [194 marks]

1. [Maximum mark: 12] 26M.1.SL.TZ1.7

Consider the function $f(x) = \frac{2x-3}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$.

The graph of f and its asymptotes are shown in the following diagram.



(a) Find the equation of

(a.i) the vertical asymptote; [1]

(a.ii) the horizontal asymptote. [2]

(b) Find the coordinates of the points where the graph of f intersects the coordinate axes. [4]

Consider the function $g(x) = x + 4$, $x \in \mathbb{R}$.

(c.i) Describe the transformation that transforms the graph of $y = f(x)$ to the graph of $y = (g \circ f)(x)$. [3]

(c.ii) Hence, find the range of $g \circ f$. [2]

2. [Maximum mark: 14] 26M.1.SL.TZ2.7

The function f is defined by $f(x) = \frac{4x-9}{2x-5}$, where $x \in \mathbb{R}, x \neq \frac{5}{2}$.

(a) Find the value of

(a.i) the y-intercept; [1]

(a.ii) the x-intercept. [1]

(b) Find the equations of all the asymptotes of the graph of $y = f(x)$. [2]

(c) Sketch the graph of $y = f(x)$, clearly showing all features found in parts (a) and (b). [2]

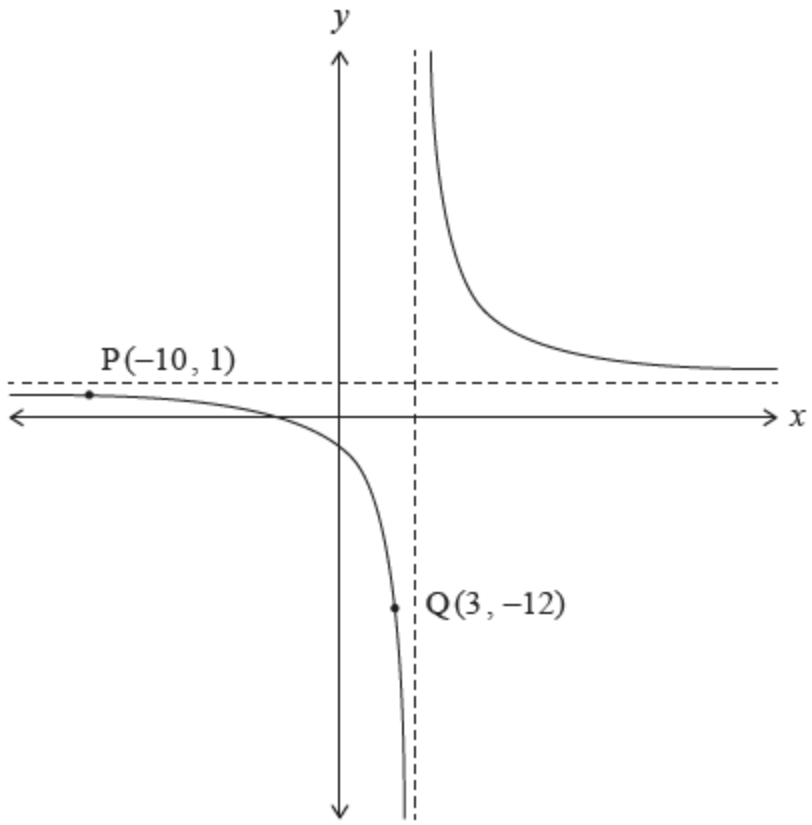
(d) Find the inverse function, $f^{-1}(x)$, and state its domain. [4]

(e) Hence, find the value(s) of x that satisfy $f(x) = f^{-1}(x)$. [4]

3. [Maximum mark: 8] 25N.1.SL.TZ3.5

The following diagram shows the graph of $y = \frac{Ax+B}{x-4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $\mathrm{P}(-10,\ 1)$ and $\mathrm{Q}(3,\ -12)$.



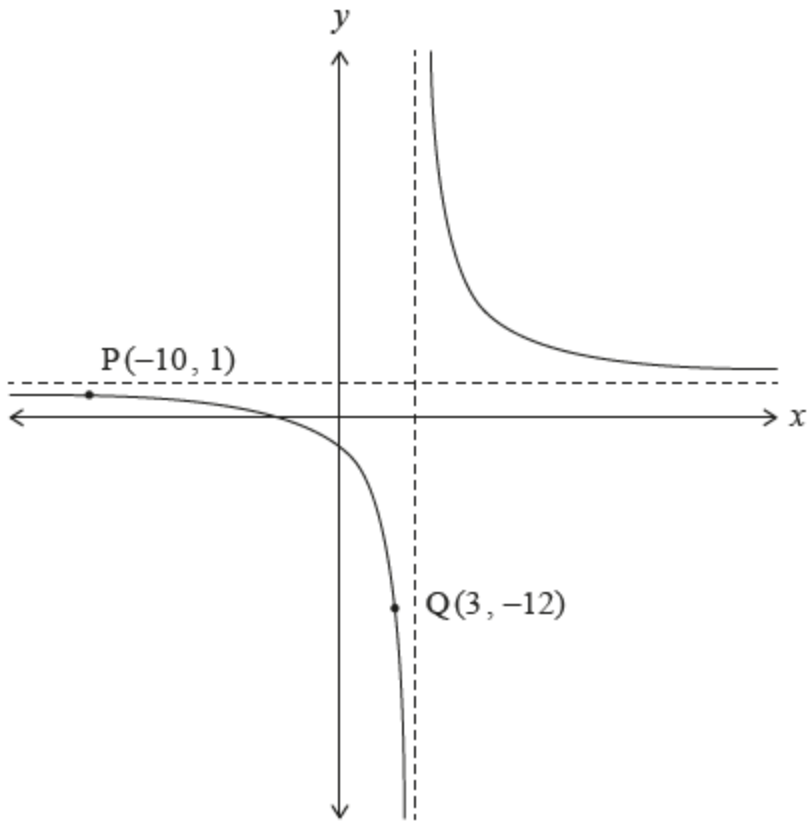
(a) Determine the value of A and the value of B. [3]

(b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax+B}{x-4}$. [5]

4. [Maximum mark: 8] 25N.1.AHL.TZ3.3

The following diagram shows the graph of $y = \frac{Ax+B}{x-4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $\mathrm{P}(-10,\ 1)$ and $\mathrm{Q}(3,\ -12)$.



(a) Determine the value of A and the value of B. [3]

(b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax+B}{x-4}$. [5]

5. [Maximum mark: 13] 25N.2.SL.TZ1.7

Consider the function $f(x) = \frac{3x+1}{2x-4}$ where $x \neq 2$.

(a) Write down the equation of

(a.i) the vertical asymptote of the graph of f; [1]

(a.ii) the horizontal asymptote of the graph of f. [1]

(b) Find $f'(x)$. [4]

(c) Hence, explain briefly why f is never increasing. [2]

The point $\mathrm{P}(x,\ f(x))$ is on the graph of f such that $x < 2$.

Let d be the distance between the points P and $\mathrm{Q}(3,\ 5)$.

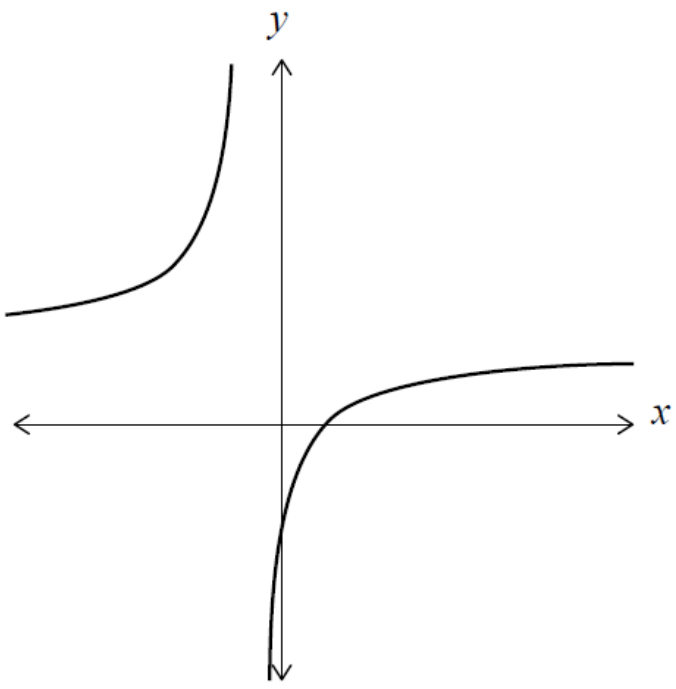
(d) Find an expression for d in terms of x. [2]

(e) Hence, find the coordinates of P such that d is a minimum. [3]

6. [Maximum mark: 5] 25M.1.SL.TZ2.3

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

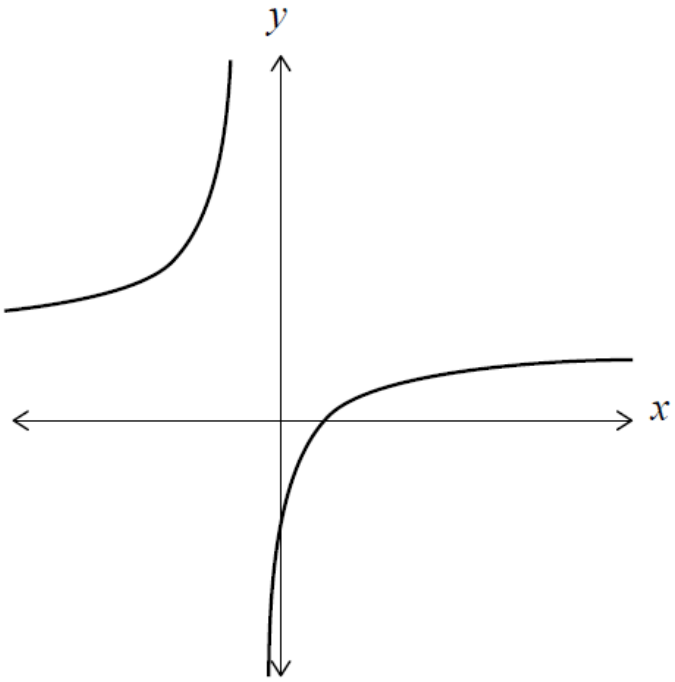
The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

7. [Maximum mark: 5] 25M.1.AHL.TZ2.1

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

8. [Maximum mark: 15] 25M.2.SL.TZ1.9

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R},\ x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $\mathrm{P}\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0,\ 0)$.

(b.i) Find the value of k. [3]

(b.ii) Hence, write down the coordinates of P. [1]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R},\ x \neq -\frac{d}{c}$, and $c,\ d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(c.i) the vertical asymptote; [1]

(c.ii) the horizontal asymptote. [1]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d. [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

9. [Maximum mark: 14] 24M.1.SL.TZ1.8

Consider the function $f(x) = \frac{4x+2}{x-2},\ x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

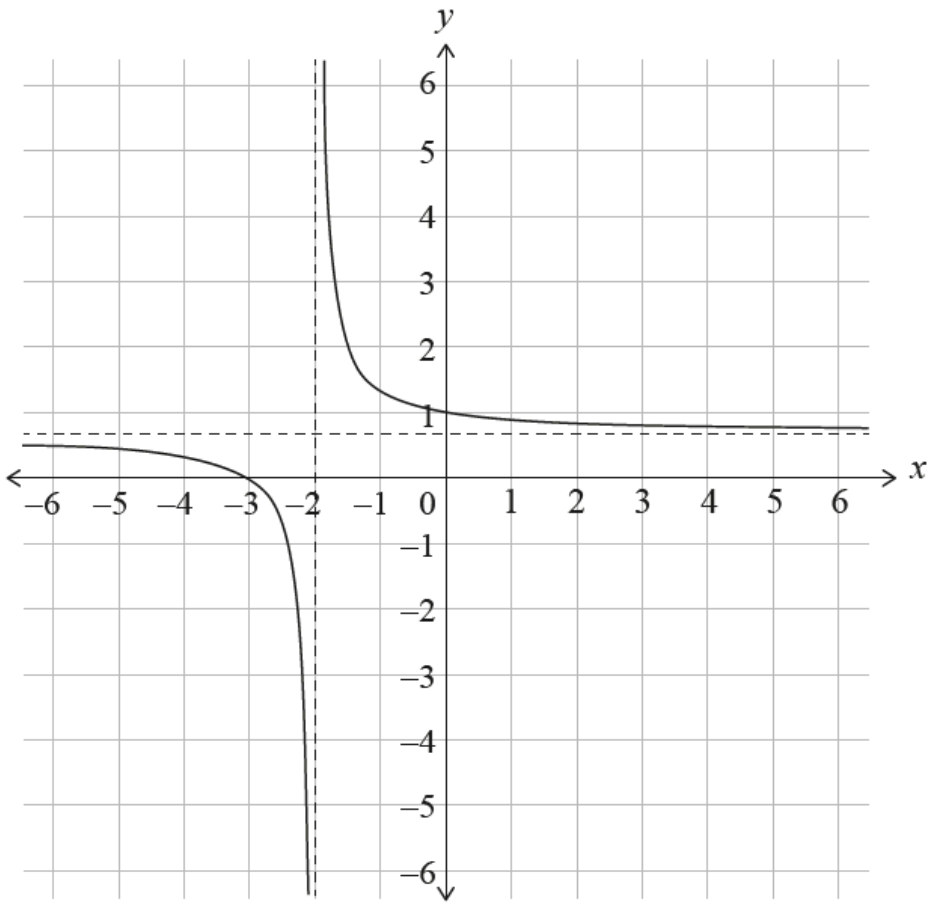
(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

10. [Maximum mark: 8] 24M.1.SL.TZ2.6

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R},\ x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R},\ m \neq 0$.

(b.i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$. [1]

(b.ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x. [4]

(b.iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [2]

11. [Maximum mark: 16] 24M.1.AHL.TZ1.10

Consider the function $f(x) = \frac{4x+2}{x-2},\ x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

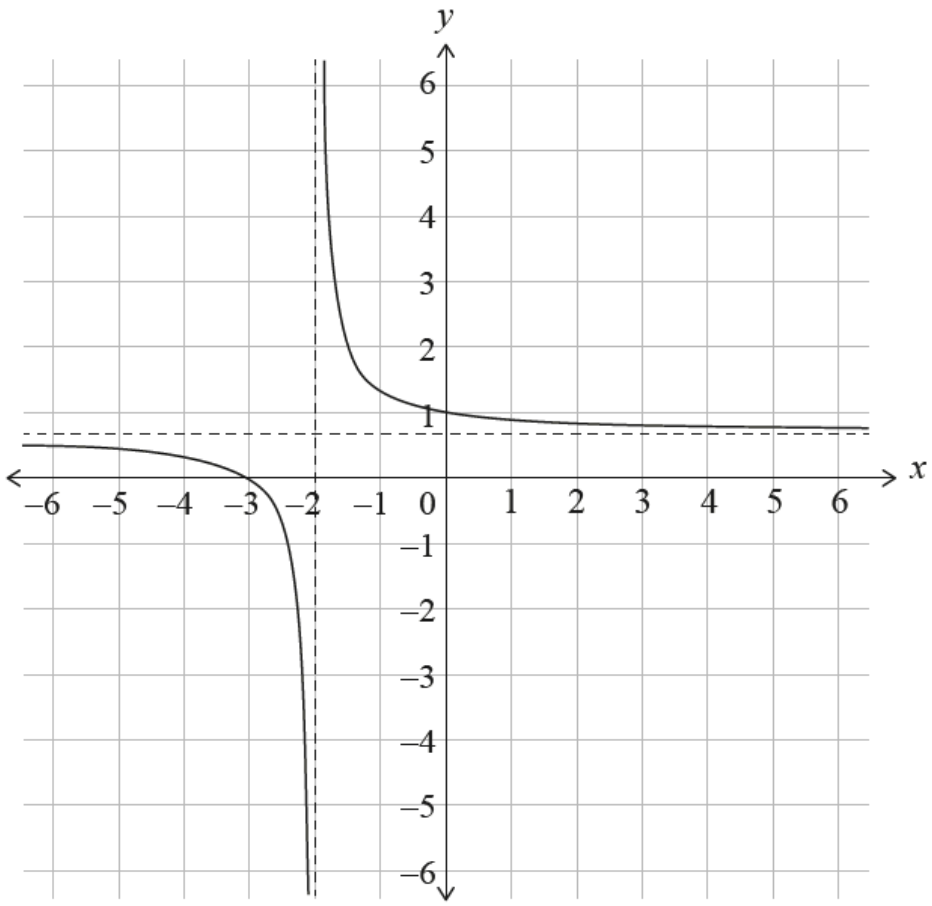
(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

12. [Maximum mark: 8] 24M.1.AHL.TZ2.5

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R},\ x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R},\ m \neq 0$.

(b.i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$. [1]

(b.ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x. [4]

(b.iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [2]

13. [Maximum mark: 7] 23M.1.SL.TZ1.2

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

(b.ii) the horizontal asymptote. [1]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

14. [Maximum mark: 5] 23M.1.SL.TZ2.3

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}, x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

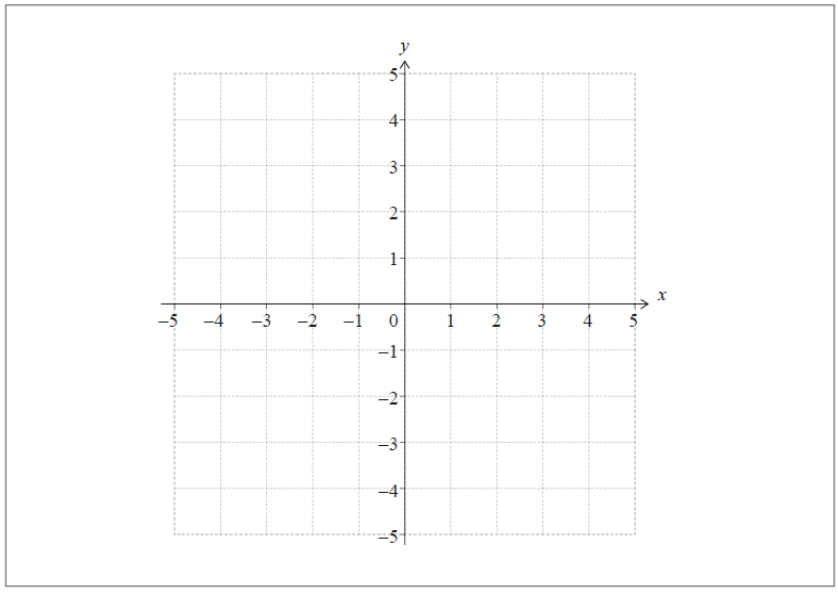
(a.ii) the horizontal asymptote. [1]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

(b.ii) the x-axis. [1]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



[1]

15. [Maximum mark: 7] 23M.1.AHL.TZ1.1

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

(b.ii) the horizontal asymptote. [1]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

16. [Maximum mark: 5] 23M.1.AHL.TZ2.2

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}, x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

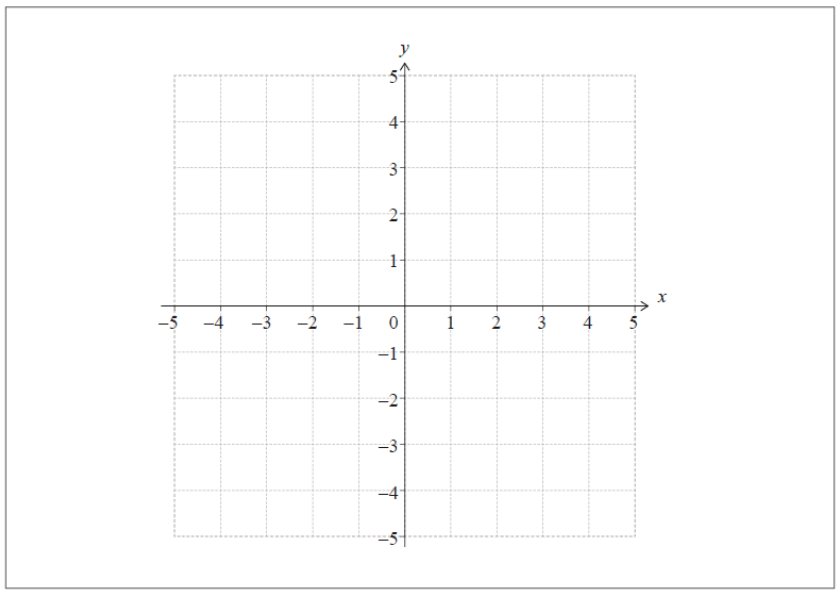
(a.ii) the horizontal asymptote. [1]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

(b.ii) the x-axis. [1]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



[1]

17. [Maximum mark: 6] 22M.1.SL.TZ2.4

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

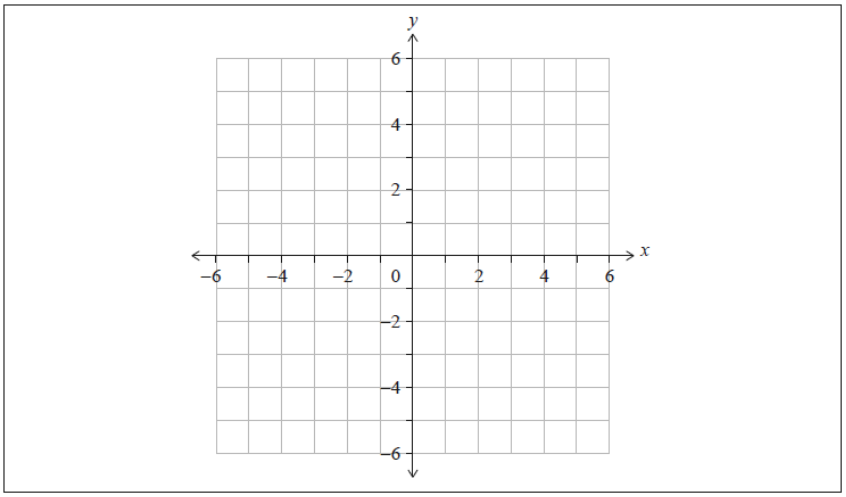
The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

(a.ii) Write down the equation of the horizontal asymptote. [1]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

18. [Maximum mark: 8] 22M.1.AHL.TZ2.3

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

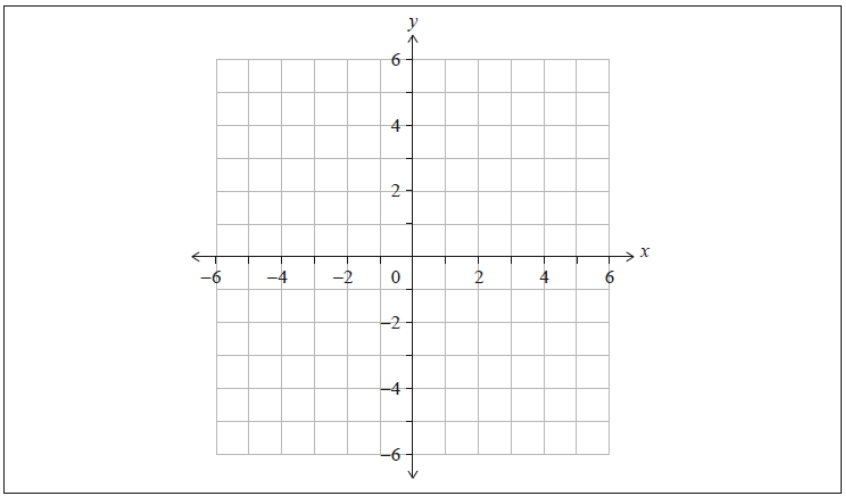
The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

(a.ii) Write down the equation of the horizontal asymptote. [1]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

(d) Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. [2]

19. [Maximum mark: 16] 22M.2.SL.TZ1.8

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R},\ x \neq -4$.

For the graph of f

(a.i) write down the equation of the vertical asymptote. [1]

(a.ii) find the equation of the horizontal asymptote. [2]

(b.i) Find $f^{-1}(x)$. [4]

(b.ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [4]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c.i) Find the value of p and the value of q. [2]

(c.ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [3]

20. [Maximum mark: 5] 21N.1.SL.TZ0.3

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

(a.i) the vertical asymptote of the graph of f. [1]

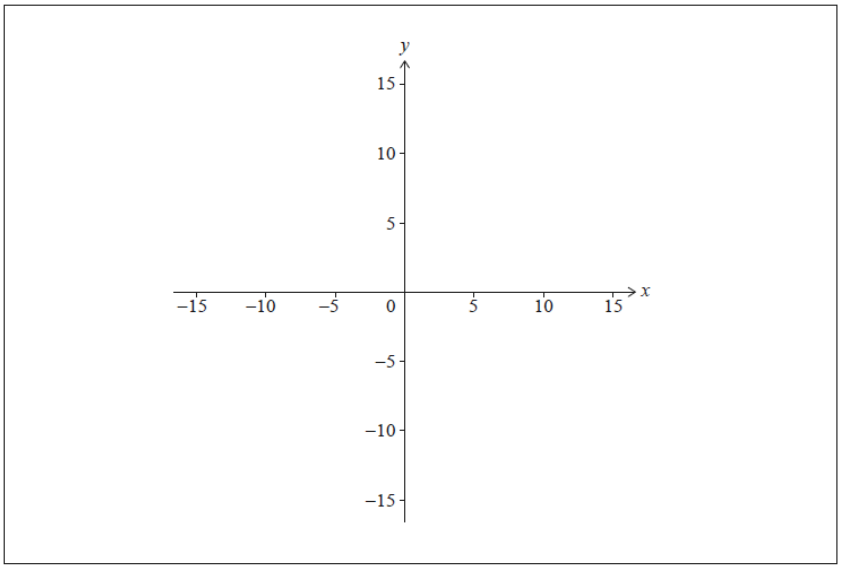
(a.ii) the horizontal asymptote of the graph of f. [1]

Find the coordinates where the graph of f crosses

(b.i) the x-axis. [1]

(b.ii) the y-axis. [1]

(c) Sketch the graph of f on the axes below.



[1]

21. [Maximum mark: 9] 21N.1.AHL.TZ0.2

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

(a.i) the vertical asymptote of the graph of f. [1]

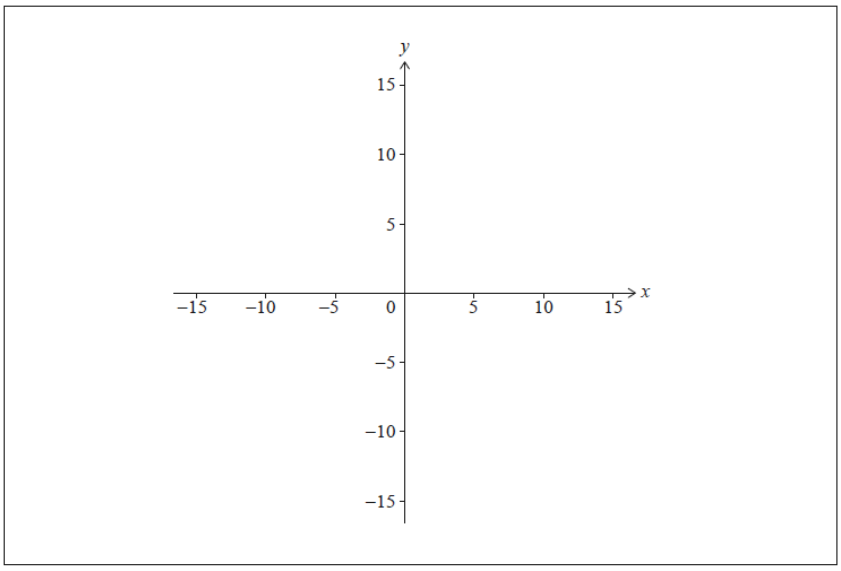
(a.ii) the horizontal asymptote of the graph of f. [1]

Find the coordinates where the graph of f crosses

(b.i) the x-axis. [1]

(b.ii) the y-axis. [1]

(c) Sketch the graph of f on the axes below.



[1]

(d) The function g is defined by $g(x) = \frac{ax+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$ and $a \in \mathbb{R}$.

Given that $g(x) = g^{-1}(x)$, determine the value of a. [4]

© International Baccalaureate Organization, 2026